INTERACTION BETWEEN ACOUSTIC WAVES AND THE BURNING SURFACE OF SOLID PROPELLANTS AT ELEVATED TEMPERATURES

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The interaction between pressure waves and a burning surface is analyzed within the framework of linear theory of acoustic instability of compacted systems. The nature of this interaction is described by the acoustic admittance of the combustion zone.

To solve the acoustic problem the model of the combustion zone must be simplified to include only the principal characteristics of the combustion mechanism [1–5]. In one of the most recent papers [5], a formula for the acoustic admittance of a burning surface was obtained with allowance for the exothermal chemical reaction in the condensed phase (k-phase). As distinct from [1, 2], it was assumed in [5] that heat release in the reaction zones is constant under nonstationary conditions. The analysis was limited to perturbation frequencies on the order of $10^4$ Hz, in which case the inertia properties of the thermal layer in the gas phase of the combustion zone need not be taken into account.

In the following, the analysis developed in [5] is applied to the case of higher frequencies, for which nonstationary processes in the thermal layer of the gas phase are appreciable. It is assumed that at frequencies up to $10^5$ Hz, the dimensions of the combustion zone in the gas are much smaller than the length of the acoustic wave and that the pressure in the combustion zone is a function only of time.

**1. Combustion-zone model and mathematical relations.** The problem is analyzed in a one-dimensional statement for a semi-infinite region. The combustion zone model is shown in Fig. 1, where (1) is the thermal layer in the k-phase free of chemical reactions; (2) is the chemical reaction zone in the k-phase; (3) is the heated zone in the gas phase, where chemical reactions may be neglected; (4) is the reaction zone in the gas; and (5) are the end products of propellant combustion.

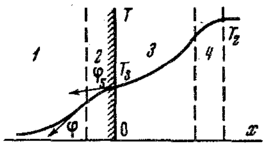


Fig. 1

Analysis of characteristic times for various zones shows [1, 3] that for frequencies on the order of $10^5$ Hz, the thermodynamic parameters and rate of burning in zones 2 and 4 are capable of responding to pressure variation, while nonstationary processes are parametrically time-dependent through the boundary conditions. Nonstationary heat- and mass-transfer equations must be used in zones (1) and (3).

We assume, as in [5], that the chemical reactions in the gas phase and k-phase occur within a narrow temperature range close to the surface temperature $T_S$ and to the isobaric flame temperature $T_2$, while the heat-release values in the reaction zones are constant and equal to $Q_1$ and $Q_2$, respectively.

Let us derive equations which relate the linear disturbances of the thermodynamic parameters to the burning-rate disturbances in regions (1)–(4).

a) Heated zone in the k-phase. Zone (1) is the most inertial zone, where heat propagation processes are described by the nonstationary heat conduction equation without heat sources. If the pressure p and the burning rate u experience small harmonic perturbations at a frequency $\omega$, then by solving the equation for the amplitude $\delta T$ of the temperature disturbances for the boundary conditions:

$$x = -\infty,\ \delta T = 0;\ x = 0,\ \delta T = \delta T_s$$

we find that

$$\beta_{1,2}=1\pm\sqrt{1+4i\Omega},\quad \Omega=\frac{\varkappa_1\omega}{u_1^2},\quad \varphi=\frac{u_1}{\varkappa_1}(T_s-T_0) \tag{1.1}$$

where $\varkappa_1$ is the thermal diffusivity of the k-phase, and

$$\delta T(x)=\left(\delta T_s-i\frac{\varphi}{\omega}\delta u_1\right)\exp\left(\frac{u_1\beta_1}{2\varkappa_1}x\right)+i\frac{\varphi}{\omega}\delta u_1\exp\left(\frac{u_1}{\varkappa}x\right)$$

Differentiating (1.1) and setting x = 0, we obtain an equation relating the disturbance amplitudes of the mass burning rate $\delta m_1=\rho_1\delta u_1$, temperature, and temperature gradient at the right-hand boundary of the thermal layer in the k-phase

$$\frac{\delta\varphi}{\varphi}=\frac{\beta_1}{2}\frac{\delta T_s}{T_s-T_0}+i\frac{\beta_2}{2\Omega}\frac{\delta m_1}{m} \tag{1.2}$$

Here, $m=\rho_1 u_1$ is the stationary mass burning rate, and $\rho_1$ = const is the density of the k-phase.

This relation was obtained under the assumption that the extent of region (2) and the temperature difference at its boundaries are small. Region (2) is considered conditionally to be a surface that coincides with the propellant-gas interface (x = 0).

b) Reaction zone in the k-phase. The equations for the velocity front of a zero-order chemical reaction can be written in quasi-stationary approximation in the form [6]

$$\lambda_1(\varphi_s^2-\varphi^2)=-2\rho_1 Q_1\int_{T_0}^{T_s}\Phi_1(T)\,dT,\quad \lambda_1(\varphi-\varphi_s)=mQ_1 \tag{1.3}$$

where, $\varphi$ and $\varphi_s$ are the temperature gradients at the corresponding boundaries of the reaction zone, $\lambda_1$ is the thermal conductivity, and $\Phi_1$ is the reaction rate.

Linearizing (1.3), we arrive at the equality

$$(1-\mu_1)\frac{\delta m_1}{m}-z_1\frac{\delta T_s}{T_s-T_0}+\frac{\delta\varphi}{\varphi}=0\quad \left(\mu_1=\frac{Q_1}{C_{p_1}(T_s-T_0)},\quad z_1=\frac{\lambda_1\Phi_1(T_s)}{m^2C_{p_1}}\right) \tag{1.4}$$

where $C_{p1}$ is the heat capacity of the k-phase.

c) Heated zone in the gas. The relations which we shall now derive are intended to relate the disturbances in the reaction zone of the k-phase and its thermal layer to the disturbances in the reaction zone of the gas phase. The boundaries of these zones are therefore subject to analysis.

We write the heat balance equation in the form

$$m_1C_{p_1}T_s-\lambda_1\varphi=m_2C_{p_2}T_2-m_1Q_1-m_2Q_2-\Delta Q \tag{1.5}$$

where $C_{p2}$ is the heat capacity of the gas. The quantity $\Delta Q$ characterizes the amount of heat specified by the presence of a time lag in the induction zone. From the nonstationary heat conduction equation for this zone, it follows that

$$\Delta Q=\int_0^{x_1'}c_{V_2}\frac{\partial}{\partial t}(\rho_2 T)\,dx$$

where $\rho_2$ is the density of the gas mixture, and $x_1'$ is the width of the induction zone.

With the aid of the equation of state $p/\rho_2=RT$, assuming that $p=p(t)$, we obtain

$$\Delta Q=\frac{1}{\gamma-1}\frac{\partial p}{\partial t}x_1'\quad (\gamma=C_p/C_v) \tag{1.6}$$

Linearizing (1.5) with allowance for (1.6), we finally get

$$\left(\frac{1}{\tau_1}+\mu_1\right)\frac{\delta m_1}{m}-\left(\frac{1}{\tau_2}-\mu_2\right)\frac{\delta m_2}{m}-\frac{\delta\varphi}{\varphi}+\frac{\delta T_s}{T_s-T_0}-\frac{1}{\tau_2}\frac{\delta T_2}{T_2}=i\Omega\chi\frac{\delta p}{p} \tag{1.7}$$

where

$$\tau_1=\frac{T_s-T_0}{T_s}, \quad \tau_2=\frac{C_{p_1}(T_s-T_0)}{C_{p_2}T_2} \quad \mu_2=\frac{Q_2}{C_{p_1}(T_s-T_0)}, \quad \chi=\frac{x_1'mp}{(\gamma-1)\lambda_1\rho_1(T_s-T_0)}$$

From the solution of the nonstationary continuity equation in the heated zone we obtain the relation between the disturbances of the mass burning rates

$$\frac{\partial m}{\partial x}=-\frac{\partial\rho}{\partial t} \tag{1.8}$$

Linearizing (1.8) and integrating from 0 to $x_1'$, we obtain

$$\delta m_1-\delta m_2=i\omega\frac{\delta p}{\gamma R}\int_0^{x_1'}\frac{dx}{T(x)} \tag{1.9}$$

where T(x) is the stationary temperature distribution in the heated zone of the gas phase. An analytical expression for T(x) can be obtained by integrating the stationary heat conduction equation in zone (3) for the boundary conditions

$$x=0,\ T=T_s;\ x=x_1',\ T=T_2$$

As a result, we get

$$T(x)=T_s+\frac{T_2-T_s}{\exp(mc_{p_2}x_1'/\lambda)-1}\left[\exp\frac{mc_{p_2}x}{\lambda_2}-1\right]\ldots \tag{1.10}$$

Substituting (1.10) into (1.9), after some simple transformations, we obtain the equality

$$\frac{\delta m_1}{m}-\frac{\delta m_2}{m}=i\Omega\Theta\frac{\delta p}{p} \tag{1.11}$$

where

$$\Theta=\frac{mpc_1}{\rho_1\lambda_1c_{p_2}(\gamma-1)}\left(x_1'-\frac{\lambda_2}{mc_{p_2}}\ln\frac{T_2}{T_s}\right)\left(T_s-\frac{T_2-T_s}{\exp(mc_{p_2}x_2'/\lambda_2)-1}\right)^{-1}$$

The stationary heat conduction equations for the condensed and gas phases can be used to determine the width of the conduction zone $x_1'$. Integration of these equations with allowance for the discontinuity of the heat flows at the boundaries of zone (2), under the assumption that the temperature at the right-hand boundary of the induction zone differs only slightly from the flame temperature $T_2$, allows us to express $x_1'$ in the form

$$x_1'=\frac{\lambda_2}{mc_{p_2}}\ln\frac{c_{p_2}(T_2-T_s)+[c_1(T_s-T_0)-Q_1]\lambda_2/\lambda_1}{(\lambda_2/\lambda_1[c_1(T_s-T_0)-Q_1]\lambda_2/\lambda_1} \tag{1.12}$$

d) Chemical reaction zone in the gas. In this zone, the pressure and temperature dependence of the flame-front mass propagation rate $m = m(p, T_2)$ may be assumed known. In accordance with Zel'dovich and Frank-Kamenetskii's formula for small perturbations, we have

$$\frac{\delta m_2}{m}=\nu\frac{\delta p}{p}+\frac{\varepsilon}{\tau_2}\frac{\delta T_2}{T_2} \qquad \nu=\left(\frac{\partial\ln u}{\partial\ln p}\right)_{T_0}, \quad \varepsilon=\left(\frac{\partial\ln u}{\partial T_0}\right)_p(T_s-T_0) \tag{1.13}$$

2. **Acoustic admittance.** Since the length of the acoustic wave for frequencies on the order of $10^5$ Hz is greater than the width of the combustion zone by at least an order of magnitude, the front of the chemical reactions in the gas can be considered to coincide with the surface of the k-phase.

The acoustic admittance of such a surface may be written in dimensionless form as

$$\xi=-\rho c\frac{\partial u_2}{\delta p} \tag{2.1}$$

where $\rho$ and c are the gas density and the speed of sound in the gas, respectively.

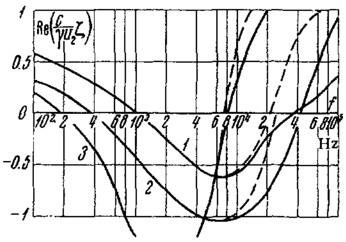


Fig. 2

In addition to the generation of an incident and a reflected wave, interaction of a pressure wave with the burning surface creates also an entropy wave which propagates at the same velocity as the gas flow [4]. It can be shown that with allowance for the inertia properties of zone (3) and the presence of the entropy wave, the formula which relates the disturbances of the gas velocity with those of the thermodynamic variables at the boundary of the combustion zone has the form

$$\frac{\delta u_2}{u}=\frac{\delta m_1}{m}+\frac{\delta T_2}{T_2}-(1+i\Omega\theta)\frac{\delta p}{p} \tag{2.2}$$

Expressing $\delta m_1$ and $\delta T_2$ through $\delta p$ with the aid of (1.2), (1.4), (1.7), (1.11), and (1.13), and substituting the relations obtained into Eq. (2.2), we obtain a relation between $\delta u_2$ and $\delta p$ with the aid of which we define the acoustic admittance in the form

$$\xi=\gamma\frac{u_2}{c}\left\{1-\nu+\nu(\varepsilon+\tau_2)\frac{1-z+\zeta(2-\mu_1)}{(1-z)\varepsilon+v}+\right.$$
$$\left.+i\Omega(\varepsilon-\tau_2)\frac{\theta(1-z)+\sigma+\varepsilon[\theta(2-\mu_1)+\sigma]}{(1-z)\varepsilon+v}\right\}\ldots,\quad z=\frac{\sqrt{1+4i\Omega}-1}{2i\Omega}$$
$$\sigma=\Theta\left(\frac{1}{\tau_2}-\mu_2\right)-\zeta\quad v=\frac{(2-\beta_1)(2\varepsilon-\varepsilon\mu_1-1)}{2(z_1-1)}-1,\quad \zeta=\frac{2-\beta_1}{2(z_1-1)} \tag{2.3}$$

Acoustic waves reflected from the burning surface are amplified if the real part of (2.3) is negative. By breaking up (2.3), we obtain

$$\mathrm{Re}\left(\xi\frac{c}{u_2\gamma}\right)=\frac{-f_1(x)+2Bbx(x^2-1)(x-x_1)}{f_2(x)+2b^2x(x^2-1)(x-x_2)}-\frac{\varepsilon+\tau_2}{2}\frac{x(x^2-1)f_3(x)}{f_2(x)+2b^2x(x^2-1)(x-x_2)} \tag{2.4}$$

where the following specifications are introduced in correspondence with [5]:

$$x=[0.5+0.5(1+16\Omega^2)^{1/2}]^{1/2}$$
$$x_1=(z_1-1)\,[(2-\mu_1)(2\varepsilon-\varepsilon\nu+\nu\tau_2)-2(2-\mu_1)(\varepsilon+\nu\tau_2)\,\varepsilon-(1-\nu)(2-\varepsilon)+$$
$$+\,\varepsilon+\nu\tau_2]\,[(2-\mu_1))(\varepsilon+\nu\tau_2)-(1-\nu)]^{-1}\,[1-(2-\mu_1)\varepsilon]^{-1}$$
$$x_2=2(\varepsilon-1)(z_1-1)\,[(2-\mu_1)\varepsilon-1]^{-1}$$
$$f_1(x)=a_1x^2+b_1x+c_1,\ f_2(x)=(1-\varepsilon)x^2+(1-\varepsilon^2)x+2\varepsilon,\qquad f_3(x)=a_3x^2+b_3x+c_3$$
$$a_1=(1-\varepsilon)[\nu(\tau_2+1)-(1-\varepsilon)],\quad a_3=(1-\mu_1)(\theta+\sigma\varepsilon)\,(z_1-1)^{-1}$$
$$b_1=\nu(1+\varepsilon\tau_2)-(1-\varepsilon^2),\ b_3=(3-\mu_1)(\theta+\sigma\varepsilon)\,(z_1-1)^{-1}$$
$$c_1=\varepsilon(\nu-2)-\nu\tau_2,\ c_3=2(\Theta+\sigma\varepsilon)$$
$$B=[(2-\mu_1)(\varepsilon+\nu\tau_2)-(1-\nu)]\,[2(z_1-1)]^{-1},\quad b=[\varepsilon(2-\mu_1)-1]\,[2(z_1-1)]^{-1}$$

Without allowance for the inertia properties of the induction zone, the formula for the acoustic admittance will contain only the first term, which coincides exactly with that obtained in [5]. The second term compensates for the inertia properties of the gas phase; it begins to have effect only at frequencies greater than $10^4$ Hz. This is clearly demonstrated in Fig. 2, where values of the acoustic admittance are shown as a function of the frequency $f$ (in Hz) in the form of the curves (1) $\{0.5, 0.9\}$, (2) $\{0.7, 0.9\}$, and (3) $\{0.5, 1.1\}$. Here, the braces show combinations of the parameters $\varepsilon$ (first number) and $\nu$ (second number); the dashed curves correspond to computations from formula (2.4) in which the second term is neglected. It can be seen that compensation for the inertia properties tends to widen the acoustic instability region at elevated frequencies.

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